AIMMS Tutorial for Professionals - Model Description

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Chapter 3

Model Description

This chapter

In this chapter you will find a description of the mathematical program corresponding to the problem description of the previous chapter.

3.1 Product flow

Indices

The following indices capture the dimensions of the problem, and are used throughout this chapter.

Indices:

l	*locations*
f	*factories* $\subset$ *locations*
c	*distribution centers* $\subset$ *locations*
p	*production lines*
t	*time periods*
s	*demand scenarios*

Decision variables

The following product flow decision variables determine the levels of production, distribution and storage.

Variables:

q_{ft}	*total factory production [hl (hectoliter)]*
u_{fpt}	*binary to indicate that production line is in use*
x_{fcts}	*transport [TL (truckload)]*
y_{lts}	*stock [hl]*

Note that the production variables are identical for all demand scenarios, while the distribution and storage variables can vary for each scenario. Note also that both hectoliters and truckloads are used to measure the quantities of soft drinks. In this tutorial a truckload is defined as 12 cubic meters.

Parameters . . .

The following product flow related parameters are used in this chapter.

Parameters:

D_{cts}	*demand [hl]*
L_t	*actual period length [day]*

Q_{fp}	*production at full operation [hl/day]*
M_{fpt}	*binary to indicate that production line is in maintenance*
V_{ft}	*binary to indicate a vacation period*
F	*drop in workforce during vacation periods (fraction)*
A_{fpt}	*potential production [hl]*
X_f	*number of available truckloads [TL]*
$\overline{Y}_l$	*maximum stock level [hl]*
$\underline{Y}_l$	*minimum stock level [hl]*

. . . and their data source

The parameters related to production line capacity, demand and vacations will be read from external data sources. The maintenance parameter will be determined as part of the rolling horizon solution process.

Potential production determination

The potential production of a production line, A_{fpt}, is dependent on the maintenance and vacation parameters, and is defined as follows.

$$A_{fpt} = L_t(1 - M_{fpt})(1 - F \cdot V_{ft})Q_{fp}, \quad \forall(f,p,t)$$

Note that nonzero values of parameters M_{fpt}, F and V_{ft} result in the potential production, A_{fpt}, being less than the production level at full operation Q_{fp}.

Balance constraint

The following stock balance constraint relates stock to previous stock, production, distribution and demand.

$$\begin{aligned} y_{lts} &= y_{l,t-1,s} + q_{lt} + \sum_f x_{flts} - \sum_c x_{lcts} - D_{lts}, \quad \forall(l,t,s) \\ y_{lts} &\in [\underline{Y}_l, \overline{Y}_l], \quad \forall(l,t,s) \end{aligned}$$

Domain restrictions

Note that this balance constraint is used for all locations (thus both factories and distribution centers), and that particular terms inside this constraint must on some occasions be interpreted as non-existent. For instance, the production term is non-existent for distribution centers, while the demand term is non-existent for factories. In AIMMS you can specify a global index domain for each identifier, and the system will automatically restrict all identifier references to such an index domain.

Factory production

Using the potential production parameter A_{fpt} as defined previously, it is now straightforward to determine the total weekly production at each of the factories.

$$q_{ft} = \sum_p A_{fpt} u_{fpt}, \quad \forall(f,t)$$

Transport limitation

It is also straightforward to model the restriction that the number of truckloads to be moved from a factory during a particular week is limited by the number of trucks available at that factory.

$$\sum_c x_{fcts} \leq X_f, \quad \forall (f,t,s)$$

Note that the above planning constraint is, in practice, a simplification of the detailed transport capacity scheduling limitations. In scheduling applications the routing of vehicles, the distances to be traveled, plus the time-windows for the drivers would all be key factors in the determination of a final schedule. These factors are considered to be less important for the current one-year plan.

3.2 Mode switches

Additional notation

The following variable is needed to register the mode switches,

Variable:

v_{fpt} *binary to register a mode switch*

Mode switch registration

The registration of mode switches seems tricky at first, but becomes straightforward with some additional explanation. Consider the following two inequalities.

$$\begin{aligned} v_{fpt} &\geq u_{fpt} - u_{fp,t-1}, \quad \forall (f,p,t) \\ v_{fpt} &\geq u_{fp,t-1} - u_{fpt}, \quad \forall (f,p,t) \end{aligned}$$

Whenever a production line switches from being used to not being used, or vice versa, the switch-registration variable v will be greater than or equal to unity. The penalty term in the objective discussed in the next section will ensure that this variable remains as small as possible. Thus, without a switch in the use of a production line, the variable v will be zero.

Effect on maintenance

Consider a production line in use. Whenever such a line needs to be maintained, its production drops to zero. Immediately following the maintenance week, its production is likely to restart. In this case, the change in production is not considered to be a mode switch. The definition of the potential production parameter, A_{fpt}, in the previous section is consistent with this observation. The maintenance parameter, M_{fpt}, is set to one when maintenance is planned, which forces the potential production parameter, A_{fpt}, to be zero for that week. The penalty term in the objective function, however, will cause the u variable to remain at level one, thus avoiding the unwanted mode switch. A similar argument applies to maintenance while a line is not in use.

3.3 Objective

The following parameters and variables are needed to specify the objective function of the mathematical program.

Additional notation

Parameters:

C_f^q	*unit production cost [$/hl]*
C_l^y	*unit stock cost [$/hl]*
C_{fc}^x	*unit transport cost [$/TL]*
C^v	*penalty cost due to mode switch [$]*
P_s	*demand scenario probability*

Variables:

r_s	*demand scenario cost [$]*
z	*total cost [$]*

The cost per single demand scenario is the sum of the production costs, the scenario-specific storage and distribution costs, plus a penalty term to reflect the costs associated with mode switching.

Cost per scenario

$$r_s = \sum_{ft} C_f^q q_{ft} + \sum_{lt} C_l^y y_{lts} + \sum_{fct} C_{fc}^x x_{fcts} + \sum_{fpt} C^v v_{fpt}, \quad \forall s$$

The total cost to be minimized is simply the weighted sum of the scenario costs.

Minimize total cost

Minimize:

$$z = \sum_s P_s r_s$$

3.4 Model summary

The full mathematical description of the optimization model can now be summarized as follows.

Minimize:

$$z = \sum_s P_s r_s$$

Subject to:

$$y_{lts} = y_{l,t-1,s} + q_{lt} + \sum_f x_{flts} - \sum_c x_{lcts} - D_{lts} \qquad \forall (l, t, s)$$

$$q_{ft} = \sum_p A_{fpt} u_{fpt} \qquad \forall (f, t)$$

$$\sum_c x_{fcts} \leq X_f \qquad \forall (f, t, s)$$

$$v_{fpt} \geq u_{fpt} - u_{fp,t-1} \qquad \forall (f, p, t)$$

$$v_{fpt} \geq u_{fp,t-1} - u_{fpt} \qquad \forall (f, p, t)$$

$$r_s = \sum_{ft} C^q_f q_{ft} + \sum_{lt} C^y_l y_{lts} + \sum_{fct} C^x_{fc} x_{fcts} + \sum_{fpt} C^v v_{fpt} \qquad \forall s$$

$$u_{fpt} \in \{0, 1\} \qquad \forall (f, p, t)$$

$$x_{fcts} \geq 0 \qquad \forall (f, c, t, s)$$

$$y_{lts} \in [\underline{Y}_l, \overline{Y}_l] \qquad \forall (l, t, s)$$

$$v_{fpt} \geq 0 \qquad \forall (f, p, t)$$